Combinatorics (MATH 180A) - Fall 2019

Instructor: Olivier Bernardi

Instructor: Olivier Bernardi – Goldsmith 301 – bernardi@brandeis.edu

Course description: This class will cover several topics in Combinatorics with an emphasis on enumerative combinatorics. It is intended for graduate students, and for very motivated undergraduate students.

Combinatorics (a.k.a. discrete mathematics) is the branch of mathematics concerned with the study of finite structures, such as graphs, lattice paths, finite groups, etc. Combinatorialists try to uncover hidden patterns in these structures, in order to answer questions such as:

- do these structures decompose into simpler components?
- can these structures be given an alternative description?
- is there a simple characterization of these structures?
- is there a simple formula for the number of such structures?

In this class we shall cover the following subjects:

- Basic combinatorial structures, and basic counting techniques and results. Sets, graphs, trees, paths, permutations, and partitions. Bijective counting, combinatorial identities.
- Generating functions and their applications to counting graphs, paths, permutations, and partitions. Lagrange inversion formula and exponential formula. Asymptotic enumeration.
- Some elements of graph theory: coloring, orientations, spanning trees, graph invariants, min-cut max-flow theorem and related results.

If there is additional time, we may cover some additional subjects such as enumeration under group action, poset theory, or introduction to symmetric functions.

Learning goals: The main goal is to get some practice in dealing with common combinatorial structures which are pervasive in many branches of mathematics (such as probability and algebra) and in other scientific fields (such as computer science and mathematical physics). In particular, we shall get familiar with the basic combinatorial structures which form the
basic blocks of many more complex ones, we shall learn how to detect recursive decompositions of combinatorial structures, and we shall learn how to translate decomposition properties into enumerative results.

**Requirement:** The class is open to any graduate student, and to highly motivated undergraduate students having already completed linear algebra (MATH15 or MATH22a), Math23a, Math28a or equivalent, and Math36a or Math39. Please be aware that this will be a fast-paced class.

**Textbook:** We will not follow a particular book, but good references are Richard Stanley’s *Enumerative combinatorics Volume 1 and 2* (Chapter 1, 5, and 7), Flajolet and Sedgewick’s *Analytic combinatorics* (Chapters 1,2), and Bollobas’ *Modern graph theory* (Chapters 1,2,3 and 5).

**Grading Policy:** The grade will be based on the biweekly homework assignments.

**Office hours:** Office hours will be held in Goldsmith 301 on Fridays 3.00pm-5.00pm, or by appointment.

**Expectation of students’ effort:** Success in this course is based on the expectation that students will spend a minimum of 9 hours of study time per week in preparation for the classes (reviewing class material, completing homeworks).

**Disabilities:** If you are a student with a documented disability on record at Brandeis University and wish to have a reasonable accommodation made for you in this class, please see me immediately.

**Academic Integrity:** You are expected to be familiar with, and to follow, the University’s policies on academic integrity. Please consult Brandeis University Rights and Responsibilities for all policies and procedures. All policies related to academic integrity apply to in-class and take home projects, assignments, exams, and quizzes. Students may only collaborate on assignments with permission from the instructor. Allegations of alleged academic dishonesty will be forwarded to the Director of Academic Integrity. Sanctions for academic dishonesty can include failing grades and/or suspension from the university.