Algebra II (MATH 131B) - Spring 2017

Instructor: Olivier Bernardi

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Course description: This class will continue and complete the foundation of abstract algebra. It is a continuation of the class Algebra I (MATH131A) taught by Professor Bellaiche in the Fall 2016. We will cover the following subjects:

• Reminders about rings, algebras, and modules. Multilinear algebra, and tensor product.
• Commutative algebra, with a focus on algebraic number theory (factorizations of ideals, etc.).
• Representations theory (for general algebras, but with a special focus on group algebras).

If there is additional time, we will cover some aspects of homological algebra. A more detailed course plan (subject to changes) is available below.

Learning goals: The goal is to learn fundamental tools and results of abstract algebra, which are of wide applications in many branches of mathematics (e.g. algebraic geometry, number theory, topology, algebraic combinatorics).

Requirement: Algebra I (MATH131A).

Textbook: The textbooks are not mandatory for this class, but can certainly be helpful. Good references are Algebra by Lang (for rings, modules and tensor product), Introduction to commutative algebra by Atiyah and MacDonald (for commutative algebra), and Introduction to representation theory by Etingof et al. (for representation theory).

Grading Policy: The grade will be based on the weekly homework assignments.
Office hours: Office hours will be held in Goldsmith 301 on Thursdays 1.30pm-2.30pm, or by appointment.

Expectation of students’ effort: Success in this course is based on the expectation that students will spend a minimum of 9 hours of study time per week in preparation for the classes (reviewing class material, completing homeworks).

Disabilities: If you are a student with a documented disability on record at Brandeis University and wish to have a reasonable accommodation made for you in this class, please see me immediately.

Academic Integrity: You are expected to be familiar with, and to follow, the University’s policies on academic integrity. Please consult Brandeis University Rights and Responsibilities for all policies and procedures. All policies related to academic integrity apply to in-class and take home projects, assignments, exams, and quizzes. Students may only collaborate on assignments with permission from the instructor. Allegations of alleged academic dishonesty will be forwarded to the Director of Academic Integrity. Sanctions for academic dishonesty can include failing grades and/or suspension from the university.
Course plan

The following course plan gives a more detailed overview of the topics to be covered (this is still subject to change).

1  Rings and Modules – Reviewing and completing the material seen in Algebra 1

1. **Basic concepts about rings: review.** Rings, homomorphisms, category of rings, subrings and ideals, quotients, first isomorphism theorem. Type of rings. 2. **Ideals and commutative rings.** Operations on ideals. Prime and max ideals. Chinese remainder theorem.

3. **Basic concepts about modules: review.** \(R\)-modules, submodule, homomorphism of modules, category of \(R\)-modules, prime rings, algebras. Sums of modules. Free modules and their bases.

4. **Multilinear maps.** Definition, dual module, bilinear maps. Isomorphism \(L(M, N; R) \cong L(M, R^*)\) etc). Non-degenerate, non-singular forms. Matrix form.


6. **Extension of the basis.** Restriction and extension of scalars.

2  Commutative Algebra

Reference: Atiyah and MacDonalds *Introduction to commutative algebras*

1. **Localization (part of chapter 1, 2, 3).** Multiplicative sets and equivalences of quotients. Field of fractions. Universal property. Local rings and localizations. Local properties for rings and modules.

2. **Radical, extensions and contraction of ideals (part of chapter 1, 2, 3).** Definitions, composition of operations, correspondence between prime ideals of a S-contraction and ideals disjoint from S.

3. **Primary decompositions of ideals (chapter 4 + first half of chapter 7).** Primary ideals, radical of primary ideals, primary decomposition of ideals, 1st uniqueness theorem, existence of primary decomposition for Noetherian rings.

4. **Nakayama lemma and integral dependence (part of chapter 2 + first half of chapter 5).** Nakayama lemma, equivalent definition of integral elements in a ring extension, integral closure and localization, going-up and going-down theorems.
5. Dedekind domains and discrete valuations (second half of chapter 5 and first half of chapter 9). Definition of Dedekind domains, proof that the ring of integer $\mathcal{R}$ of any algebraic number field is a Dedekind domain, factorization of ideals in primary ideals in Noetherian domains of dim 1, discrete valuations, equivalent characterization of discrete valuation rings, factorization of of ideals in prime ideals in Dedekind domains.

3 Representation theory

1. Algebras and representations. Examples and definitions
2. Decomposition of representations, and Schur lemma.
3. Representations of finite groups: fundamental isomorphism. Decomposition of the regular representation, isomorphism of algebra $\mathbb{C}[G] \simeq \bigoplus_i \text{End}(V_i)$.
5. Representations of finite groups: restricted and induced representations.
7. Fundamental isomorphism for finite dimensional algebras.
8. Filtrations of representations.