What is this course about? Linear algebra is one of the most powerful and beautiful subjects in mathematics. It offers great insight into a myriad of disciplines, and has applications ranging all the way from economic theory to image compression to ecology. But beyond its great applicability, linear algebra is characterized largely by its elegance. This class, more than anything, will give you a taste of the power of abstraction to take a concrete problem and illuminate its underlying structure.

In this class, we will start with a tangible mathematical problem - how to find all solutions to a linear system of equations - and examine it in a new light.

Learning Goals for Math 15a. Students in Math 15a will:

- Gain foundational knowledge for a broad variety of math and science courses, which they will draw on for their entire college career and beyond.
- Learn to think critically and logically about (mathematical) problems they encounter, both in this and other courses.
- Apply familiar techniques in unfamiliar settings, by relying on experience, intuition, and understanding.

Linear algebra is a challenging course. It’s worth saying up front that for many students this is a challenging course. It is the first exposure to abstract mathematics and reasoning, and switches away from the traditionally computation-heavy classes like Calculus. We will basically only learn two computational techniques in this course, one of them in the first week, but the point of the course is to keep rephrasing new questions we’re interested in in ways which can be answered by these techniques. Don’t be discouraged if it doesn’t make sense right away, it is only in struggling that we really learn!

Office Hours. Many students don’t understand what office hours are for. Office hours are not just for struggling with the course, but for learning more about the material and getting to know your professor. I strongly encourage every student to come to my office hours at least once in the semester, even if it is only to say hi and chat for a few minutes. If you would like to meet with me but cannot make the scheduled office hours, just send me an email and we will find a time to meet.

The book can be purchased from the University Bookstore, or bought online, but you are responsible for making sure you purchase the correct book. If you buy an older edition, it is your responsibility to make sure you’re reading the correct sections and doing the correct homework problems. I strongly recommend you try for a new or used version of this edition.

Prerequisites. MATH 5a and permission of the instructor, placement by examination, or any mathematics course numbered 10 or above. Students may take MATH 15a or 22a for credit, but
not both. If you HAVE NOT taken Math 10a and 10b, please come meet with me at the beginning of the semester.

Solving linear systems is a basic component of the mathematical tools used by many sciences, e.g. computer science, physics, biology, chemistry, economics, etc. In this course, students will learn the language and techniques needed to solve such systems, and extend the theory to more sophisticated settings. For example, in Calculus we can understand a lot about a function by studying its tangent line at a point. For multivariable functions, e.g. a function \( F : \mathbb{R}^2 \to \mathbb{R}^3 \), we instead have a \( 3 \times 2 \) matrix of partial derivatives, which can be viewed as a linear map that approximates the original function at a point.

**Exams.** There will be two midterm exams and a final exam.

- Exam 1:
- Exam 2:
- Final Exam: Determined by registrar block schedule.

**Grades.** Your grade in the course will be based on the following:

1. **Homework (20% of your grade)**
   - Homework assignments will be collected once a week. It will be due every Thursday at the beginning of class.
   - **No late homework will be accepted,** but your two lowest homework grades will be dropped.
   - I encourage you to discuss homework problems with your classmates, but you must write up your own solutions.

2. **Quizzes (10% of your grade)**
   - Short quizzes will be given regularly. These will be every Monday at the beginning of class.
   - **No make-up quizzes will be given.** Missed quizzes count as zeroes. However, the two lowest of your quiz grades will be dropped.

3. Two midterm exams (each 20% of your grade)
4. Final exam (30% of your grade)

**Calculators.** Calculators are not allowed during exams or quizzes. In particular, you do not need a graphing calculator. You may use a calculator when working on homework; however, please keep in mind that simply entering a problem into a computer algebra system and recording its answer does not constitute a solution to that problem.

**LATTE.** All course materials for Math 15a will be available online on LATTE. Log in at http://latte.brandeis.edu using your Unet username and password.

**Office hours.** You are encouraged to use my office hours and/or the course assistants’ office hours whenever you have questions about the course material. If you can’t attend my office hours, don’t hesitate to ask for an appointment at another time.

**Four-Credit Course (with three hours of class time each week).** Success in this 4 credit hour course is based on the expectation that students will spend a minimum of 9 hours of study time per week in preparation for class (readings, papers, discussion sections, preparation for exams, etc).
**Students with disabilities.** Brandeis seeks to welcome and include all students. If you are a student who needs accommodations as outlined in an accommodations letter, please talk with me and present your letter of accommodation as soon as you can. In order to provide test accommodations, I need the letter more than 48 hours in advance. I am happy to arrange for your accommodations, but cannot do so retroactively. If you have questions about documenting a disability or requesting accommodations, please contact Student Accessibility Support (SAS) at 781.736.3470 or access@brandeis.edu.

**Academic Integrity.** You are expected to be honest in all of your academic work. Please consult Brandeis University Rights and Responsibilities for all policies and procedures related to academic integrity. Students may be required to submit work to TurnItIn.com software to verify originality. Allegations of alleged academic dishonesty will be forwarded to the Director of Academic Integrity. Sanctions for academic dishonesty can include failing grades and/or suspension from the university. Citation and research assistance can be found at LTS - Library guides.

**Privacy.** This class may require the use of tools that may disclose your coursework and identity to parties outside the class. To protect your privacy you may choose to use a pseudonym/alias rather than your name in submitting such work. You must share the pseudonym/alias with me and any teaching assistants as needed. Alternatively, with prior consultation, you may submit such work directly to me.

**Tentative Schedule.**

**August**

Wed. 28. §1.1 Systems of linear equations.

Thu. 29. §1.2 Row reduction and echelon forms.

**September**

Wed. 4. §1.3-1.4 Vector and matrix equations.

Thu. 5. §1.5 Solution sets to matrix equations.

Mon. 9. §1.6 & 1.10 Some applications.

Wed. 11. §1.7 Linear independence.

Thu. 12. §1.8-1.9 Linear transformations and matrices.

Mon. 16. §2.1 Matrix operations and row reduction, again.

Wed. 18. §2.2-2.3 Invertibility of matrices.

Thu. 19. §4.1 Vector spaces, subspaces, and quotient spaces.

Mon. 23. §4.2 Null and column spaces.

Wed. 25. §4.3-4.4 Spanning sets and bases.

Thu. 26. §4.5 Dimension.

**October**

Wed. 2. §4.6 Rank.

Thu. 3. §4.7 Change of basis and similarity.

Mon. 7. §4.8-4.9 Some fun applications.
Thu. 10. **Midterm I**

Tue. 15. §3.1 Determinants of simple matrices.

Wed. 16. §3.2 Properties of determinants.

Thu. 17. §3.3 A formula and its interpretations.

Wed. 23. Permutations and the determinant.

Thu. 24. §5.1-5.2 Eigenvectors, eigenvalues, and the characteristic polynomial.

Mon. 28. Digression: polynomials, complex numbers, and the fundamental theorem of algebra.

Wed. 30. §5.3-5.4 Diagonalization and linear transformations.

Thu. 31. §5.5 Complex eigenvalues of real matrices.

**November**

Mon. 4. §5.6-5.7 Even more applications.

Wed. 6. Digression: the Perron-Frobenius theorem (possible catchup day).

Thu. 7. **Midterm II**

Tue. 11. Diagonalization over the complex numbers and some strange examples.

Wed. 13. §6.1-6.3 Inner product, length, and orthogonality.


Mon. 18. §6.5-6.6 Least squares.

Wed. 20. §6.7 & 7.2 Inner product spaces, quadratic forms, and symmetric matrices.

Thu. 21. §7.1 Orthogonal matrices and diagonalization of symmetric matrices.

Mon. 25. More on the spectral theorem.

**December**

Mon. 2. §7.4 Singular value decomposition.

Wed. 4. §7.5 Applications of SVD: PCA and image compression.

Thu. 5. Some applications in higher math (possible catchup day).

Mon. 9. Review for final.