Math 28b: Rings and Fields

SYLLABUS

Our book will be the free online book
Abstract Algebra: Theory and Applications

There will be weekly quiz/homework. When I give you the quiz/HW, you will get 15
minutes to answer a few questions in class (maybe two short answers). The rest you do for
homework, including those you did in class if you think you made a mistake.

Class participation will also be included in these quiz/HWs: a student who answers a
question in class may write up his answer as a topic on his quiz that week. However, the
answer should be written in complete sentences, with explanations.

There will be a Final Exam with similar rules: at the end of the course, you will get a
Take Home Final in class. You need to answer several questions in class and the rest after
class.

Rules: Quizzes and Exams are individual work. You are not allowed to communicate
with other people or AI. On Homework and Take Home part of the exam, you may col-
laborate with other students in the class and you may ask for hints from teachers. But
students must hand in separate answers in their own words.

Each student is also required to give one presentation of a homework problem or extra
topic. This should be in class. However, if we run out of time, or if there are students who
are very shy, you can do the presentation after class just to me.

Grade:

30% Quizzes
40% Homework
8% Final Exam (in class part)
12% Final Exam (take-home part)
10% Student presentation/ other class participation

Most books on Abstract Algebra, such as our textbook, begin with Groups since groups
are more basic objects than Rings and Fields. However, many students find rings and fields
to be easier concepts than groups since they are more familiar, less abstract objects. For
example,
\[ \mathbb{Z} = \{\cdots, -3, -2, -1, 0, 1, 2, 3, \cdots \} \], the ring of integers.
\[ \mathbb{R} \], the field of real numbers.
\[ \mathbb{R}[x] \], the ring of polynomials in one variable.
\[ \mathbb{C} \], the field of complex numbers.

This course will concentrate on the example which make the subject of Rings and Fields
accessible. We will start with \( \mathbb{Z} \), the ring of integers and \( \mathbb{Z}/n \) the integers modulo \( n \).
Equivalence Relations and the Division Algorithm will be the first topics of study. These
are the first two chapters of our book. Then we skip half the book and go to Chapter 16
(Rings). We have to discuss the definition of a Group since the book keeps talking about
it. For us, the key example of a group is \( \mathbb{Z}/n \). But this is also an example of a “quotient
ring” and we take this ring-theory point of view.
Most of our rings will be “commutative rings” which means \( ab = ba \). But students already know a “noncommutative” ring since matrix multiplication is noncommutative:

\[
\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} \neq \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}
\]

Instructor:
Kiyoshi Igusa
Goldsmith 305
Office hours: M 1-2:30, W 10:30-12.

1. Topics

**Preliminaries** (Chapters 1,2)
(1) Sets and Equivalence Relations
(2) Mathematical Induction
(3) Division Algorithm
(4) \( \mathbb{Z}/n \) Integers mod \( n \). (but STOP when you get to “symmetries”)

**Rings** (Chapters 16,17,18)
(1) Rings: Definition These rings might not have 1.
(2) Subring
(3) (Integral) Domains and Fields: \( \mathbb{Z}[i] \) is the first example of a Domain.
(4) Homomorphisms and Ideals
(5) Maximal and Prime Ideals
(6) Chinese Remainder Theorem
(7) Polynomial rings
(8) Fraction Fields
(9) Factorization, UFDs, PIDs, Euclidean Domains

We will skip Chapters 18,19,20

**Fields** (Chapters 21,22)
(1) Field extensions
(2) Splitting Fields
(3) Constructibility and field extensions
(4) Finite Fields
(5) Polynomial Codes We need to go back to:
(6) Error-correcting codes which use the “group” \( \mathbb{Z}/n \).

This should be no problem since \( \mathbb{Z}/n \) is a ring that we will study thoroughly throughout the course.

(7) Other topics as time permits.

Attendance is required (Dont miss two classes in a row!) and counts as part of the grade. However, classes will be recorded and missed class can be reviewed online.

Students with disability: If you are a student with a documented disability at Brandeis University and if you wish to request a reasonable accommodation for this class, please see the instructor immediately.