Course Description

Graph Theory is a branch of mathematics that encapsulates pairwise relationships. In this course, we explore this abstract landscape using real life puzzles as our landmarks. Here are a few examples:

(a) Can you swap the positions of the white and black knights using only the L shaped move of a typical chess game?

(b) Can you draw lines connecting each of the three houses to each of the three utilities without allowing the lines to cross?

(c) In the classic game Monopoly, which spaces are most frequently visited?

These are examples of puzzles that can be solved using graph theory. In mathematics, a graph is a diagram with dots (called vertices) that are connected by lines (called edges). Graphs are an important part of many areas of mathematics including combinatorics, group theory, and topology. Graphs are also used to model pairwise connections and so have a wide range of applications in computer science, biology, and network science.

In this class we will study graph theory through the lens of puzzles. In each unit, we will first try to solve some puzzles that exhibit the types of questions we want to discuss. We will then learn the graph theory behind the solutions to these puzzles and finally we will use this graph theory to solve harder puzzles or answer harder questions.

This course is designed to maximize active learning and collaboration. While there will be some lecture, a large portion of the course will involve students working in teams to practice solving puzzles and to discover graph theory through inquiry and experimentation. The main goal is to practice translating everyday problems into abstract ideas and to practice clear and rigorous communication about complex concepts.

The content of the course will be divided into 4 parts. In the first part, we will introduce basic graph theory vocabulary and learn techniques to rigorously explain the solution to a puzzle. The second part will cover traversals and path finding algorithms. This answers the question: supposing

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1One of these puzzles has no solution, one has a solution you might come up with on your own, and one should be solved using a computer. In this class we will discuss examples of puzzles in each of these three categories.
the vertices of the graph were destinations and the edges were roads, what restrictions are there on how we travel around the graph? The third part of the class will focus on the topology of the graph. This answers questions such as: can a graph be drawn on the piece of paper so the edges don’t cross? or how many colors do you need to assign each vertex a color so that no adjacent vertices have the same color? The fourth part of the course will cover optimization using graphs, including using Markov Chains to identify the optimal strategy for games that can be modeled using graphs.

Learning Goals Successful students will...

• Convert simply phrased problems into abstract ideas.
• Collaborate with others and explain technical concepts to their collaborators.
• Reason precisely and clearly about puzzles and games, especially about the existence and uniqueness of a solution to a puzzles.

Content Goals Upon successful completion of this course, students will be able to:

• Recognize the salient vocabulary of Graph Theory (e.g. vertices, edges, bipartite, isomorphism, degree, walk, cycles, trees).
• Categorize graphs according to their isomorphism classes, and justify when two graphs are or are not isomorphic.
• Identify spanning trees within a graph.
• Analyze path finding algorithms, especially Dijkstra’s algorithm.
• Distinguish between Euler and Hamilton paths and justify the conditions under which a graph may have such a path.
• Use Euler characteristic to determine when a graph has a planar embedding.
• Use induction and deletion and contraction of edges to reason about a graphs.
• Find a proper vertex coloring of a graph and determine if such a coloring is minimal.
• Find a matching of a bipartite graph.
• Use Markov Chains to find the most visited vertex in a random walk on a graph.

Textbook We will use Graph Theory: A Problem Oriented Approach by Daniel Marcus, 2nd Edition. The textbook is required as problems from it will be assigned as part of the homework.

Schedule

This is an approximation and is subject to change. Based on a semester with 39, 50 min class periods. The final projects will be presented during the final exam period.
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<th>Time</th>
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<td>Introduction</td>
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<td>Basic Graphs</td>
<td>A and B</td>
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<td>Reasoning</td>
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<td>Planar graphs</td>
<td>H</td>
<td>3 houses &amp; 3 utilities</td>
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<td>Independence and Colors</td>
<td>I, K</td>
<td>Sudoku, Map colorings</td>
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<tr>
<td>Matchings</td>
<td>C, M, N</td>
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<td>Markov Chains</td>
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<td>(as available)</td>
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**Assessments**

Your grade in the course will be based on the following:

1. **Class Participation and Teamwork (15% of your grade)** Because this is a team-based course and your team is relying on you, your attendance and class participation on days when we do team work will be recorded and incorporated into your grade.

2. **Problem Sets (25% of your grade)** Each unit will be accompanied by a problem set with exercises on the graph theory content learned in that unit. NO LATE HW WILL BE ACCEPTED, but your lowest homework score will be dropped. If you need to miss class or cannot complete an assignment, please discuss your individual situation with an instructor as soon as possible.

3. **Quizzes (15% of your grade)** There will be 4-6 short in-class quizzes throughout the semester. NO MAKE UP QUIZZES will be given, however the lowest quiz score will be dropped.

4. **Short Papers and Reflections (20% of your grade)** There will be several short writing assignments (1-2 pages) where we ask you to explain the solution to a puzzle or to reflect on the experience of solving a puzzle.

5. **Final Project (25% of your grade)** The final project will involve researching an existing puzzle (or creating a puzzle) and presenting the chosen puzzle to the class. We will also require a short (3-5 page) written statement explaining your chosen puzzle, its background, and any solutions. There will be some suggested topics for the project and a project proposal deadline midway through the semester. Students will be evaluated based on both their research and on the clarity of their final presentation.

**Course Format**

**Team-based learning** This is a Team-based Learning course. At the beginning of the course, students will be assigned a team to work with for the duration of the semester. You are not required to meet your team outside of class, but you will work with your team in class to solve puzzles. One of the main goals of our course is to for students to practice working in a team and explaining technical concepts to each other. Students will also have the option of working in groups on their final project, but this is not required.
Logistics and Policies

Prerequisites MATH 15a or equivalent course in Linear algebra is required. Alternatively, students who have taken COSI 29a (Discrete Structures) or LING 160 (Mathematical techniques for Linguistics) or similar college level discrete math course or who obtain permission from the instructor are welcome.

Students should have some familiarity with matrices and their operations. Some experience with discrete probability will also be helpful but is not required. We DO NOT require that students have taken Math 23b (Introduction to Proofs) and will instead cover some logical reasoning and proof techniques in our class. If you’re not sure if this class is right for you, please don’t hesitate to contact one of the instructors.

LATTE Throughout the semester, announcements, deadlines, assignments, etc. will be announced on LATTE, so please check it often. You must be officially registered for the course in order to access the LATTE page.

Credit Hours Expectation Success in this 4 credit hour course is based on the expectation that students will spend a minimum of 9 hours of study time per week in preparation for class (readings, papers, discussion sections, preparation for exams, etc.).

Laptops/phones in Class Laptops/phones/iPads etc are not allowed in class unless you have special permission from the instructor. There is very little reason for you to have these devices out in class and they can be distracting for your classmates. Please be polite and courteous to your peers and follow this rule.

Students With Disabilities Brandeis seeks to welcome and include all students. If you are a student who needs accommodations as outlined in an accommodations letter, please talk with me and present your letter of accommodation as soon as you can. we want to support you.

In order to provide test accommodations, we need the letter more than 48 hours in advance. we want to provide your accommodations, but cannot do so retroactively. If you have questions about documenting a disability or requesting accommodations, please contact Student Accessibility Support (SAS) at 781.736.3470 or access@brandeis.edu.

Academic Integrity You are expected to follow the University’s policy on academic integrity, which is distributed annually as section 4 of the Rights and Responsibilities Handbook. Instances of alleged dishonesty will be forwarded to the Department of Student Development and Conduct for possible referral to the Student Judicial System. Potential sanctions include failure in the course and suspension from the University. If you have any questions about how these policies apply to your conduct in this course, please ask.

Name/Pronouns If you have a preferred name and/or preferred pronouns you would like us to use, please let us know either by email or in person. The math department is committed to welcoming students of all races, genders and with a diverse backgrounds.

Disclaimer Elements of this syllabus may change as the course wears on, if we see the need to modify them.