Math 131A
Algebra

Instructor: Joël Bellaïche

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Office Hours: Tuesdays, 2pm-4pm

Class time and place: Mo-We 5:00 PM - 6:15 PM, Goldsmith Math 300

First day of class: August, Monday 29, 2016

Course Description and Objectives: The course is an introduction to the fundamental notions of abstract algebra.

Prerequisites: As the course will take each subject from scratch, no prerequisite is required except familiarity with mathematical reasoning and rudiments of Set Theory: set, functions, composition of functions, inverse and direct image of a subset, injective (a.k.a. one-to-one) and surjective (a.k.a. onto) functions. However the pace will be very fast, so having taken one or several classes of undergraduate algebra before is highly recommended.

Course Plan: The course will cover the following subjects:

1.– Groups: Basics, normal subgroups, group actions, Jordan-Hölder Theorem, subgroups of a product, p–Sylow subgroups and theorems, solvable and nilpotent group, introduction to group cohomology, extensions of groups.

2.– Rings: general rings, commutative rings (on which we will restrict our attention), polynomials over a ring, ideals and modules, principal ideal domains, noetherian rings, ring of fractions and localizations.

3.– Fields: characteristic of a field, extension of fields, finite, algebraic and transcendental extensions, normal and separable extensions.

4.– Galois theory: Galois extension and their Galois groups, the dictionary sub-extension/subgroup, Galois group of an equation, applications to solvability by radicals of equations. (If time allows, Grothendieck’s point of view on Galois theory).

In addition, the course will give an introduction to category theory (definition, examples, notions of commutative diagram, limits and co-limits, functors), between the subjects 1 and 2.

See Latte (http://lts.brandeis.edu/courses/newlatte) for a more detailed course plan where each subjects is divided in topics corresponding very roughly to one lecture.

Exercises: It is important to do regular exercises to check your understanding of the material, and to improve your skills in algebra. For each topic a set of exercises will be available on latte. It is advised that you try to do
all the exercises. Some exercises will be easy, but most will be non-trivial and should require some work. If you are stuck, try to talk with other students, or with me.

The solution of one exercise by set (i.e. by topic) must be carefully written down, and is due one week after the corresponding topic has been completely covered in class.

Evaluation: The exercises will be graded and their average will count for 35% of the final grade. There will be an in class midterm which will count for 25% of the final grade, and a final exam (in class or take-home, this will be announced later) counting for 40% of the grade.

Disabilities: If you are a student with a documented disability on record at Brandeis University and wish to have a reasonable accommodation made for you in this class, please see me immediately.

Learning goals: Students in this class will learn the basic theory of the three fundamental structures of algebra (groups, rings, fields), and be trained to manipulate them. In addition, they will be initiated to the Galois theory of equations, and will receive an initiation to the language of categories.

Work Expectation: Success in this 4-credit course is based on the expectation that students will spend a minimum of 9 hours of study time per week in preparation for class (readings, homework, preparation for exams, etc.).

Bibliography: The book shall not follow a specific book, but it is important that each student gets at least one of the following algebra textbook.
- Artin, Michael. Algebra, Pentice Hall.
- Jacobson, Basic Algebra.