Math 221a  

Topology III: Vector bundles and characteristic classes

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Meeting time: M,W,Th 1:00 PM–1:50 PM (Block F); Goldsmith 317

References:

The course will cover some basic tools in topology that arise in many other fields of mathematics, namely fiber bundles, vector bundles, and their associated characteristic classes. There are many approaches to defining and calculating characteristic classes, and I will try to give a flavor of some of them. The prerequisites are the full year of Topology (Math 151ab; through cohomology and Poincaré duality). Some knowledge of basic manifold theory (Math 140a) would be very useful as well. I will suggest exercises in the course of the lectures, and there will be a few problem sets to be handed in.

**Course outline.**

(1) Basics of fiber bundles, including basic examples such as vector bundles and principal bundles.
(2) Homotopy lifting theorem; covering homotopy theorem. Long exact sequence in homotopy.
(3) Classifying spaces.
(4) Existence and homotopy of cross-sections; basic obstruction theory.
(5) Idea of characteristic classes; axioms for Stiefel-Whitney classes.
(6) Leray-Hirsch theorem; construction of Stiefel-Whitney classes.
(7) Oriented vector bundles; the Thom isomorphism theorem; Euler class.
(8) Complex bundles and Chern classes; Pontrjagin classes.
(9) Applications to cobordism.